4.1 Notes

4.1: Divisibility

Definition: For two whole numbers a and b, $b \neq 0$, we say b divides a, written as $b \mid a$, if $a \div b$ is a whole number. Other ways to say this are "a is divisible by b", "b is a divisor of a", "a is a multiple of b", and "b is a factor of a".

Divisibility Rules: Let n be a whole number.

2 | n if and only if n ends in an even number.

3 | n if and only if the sum of the digits of n is divisible by 3.
Example: Show 3 | 5352.

4 | n if and only if 4 divides the last 2 digits of n.
Example: Show 4 | 1880.

5 | n if and only if n ends in either 0 or 5.

Example: Determine which of the numbers 2 through 11 divide 1680. Justify each of your tests.

More Examples: These problems would be good to try before your quiz.

Determine which of the numbers 2 through 11 divide the following numbers. Justify each of your tests.

(a) 6048
(b) 3300
(c) 7777
(d) 20,790

Divisibility Rules: Let n be a whole number.

6 | n if and only if 2 | n and 3 | n.

For 7, form a new number k by taking off the last digit of n and subtracting its double from the result. Then 7 | n if and only if 7 | k.
Example: Show that 7 | 3654.

8 | n if and only if 8 divides the last 3 digits of n.

9 | n if and only if the sum of the digits of n is divisible by 9.

10 | n if and only if n ends in 0.

For 11, we form a new number k by adding then subtracting the digits of n. It is important that we consider the sign of the first digit as part of this addition and subtraction. Then 11 | n if and only if 11 | k.
Example: Show that 11 | 1485.

Example: Determine which of the numbers 2 through 11 divide 13860. Justify each of your tests.

Activity: Invent divisibility rules for as many numbers less than 30 that you can find. There may be multiple rules for the same number. Warning: 13, 17, 19, 23, 27, and 29 are difficult.

Try to find at least one general rule for certain types of numbers.